JEE-Main-28-06-2022-Shift-2 (Memory Based)

Physics

Question: A body of mass 2 kg moving with speed 4 m/s encounters a region from x=0.5 to x=1.5 where F=-kx. Find the final velocity of the body. Given k=12

Options:
(a) 5 m/s
(b) 2 m/s
(c) 4 m/s
(d) 6 m/s

Answer: (b)

Solution:

$$a = -\frac{kx}{m}$$

$$v\frac{dv}{dx} = -\frac{12x}{m}$$

$$vdv = -\frac{12x}{m}dx$$

$$\left[\frac{v^2}{2}\right]_4^v = -\frac{12}{2}\left(\frac{x^2}{2}\right)_{0.5}^{1.5}$$

$$\frac{v^2}{2} - \frac{16}{2} = -6\left(\frac{2.25-0.25}{2}\right)$$

$$v^2 - 16 = -12$$

$$v^2 = -12 + 14 = 4$$

$$v = 2m/s$$

Question: A ladder rest slantly with its base 3 m from the floor The wall is frictionless. Length of ladder is √34 m Mass of ladder is 10kg. Find the ratio of reaction force by wall to reaction force by floor on ladder

Options:
(a) 3/10
(b) 9/10
(c) 5/10
(d) 7/10

Answer: (a)

Solution:

$$N_G = mg = 10 \times 10 = 100N$$

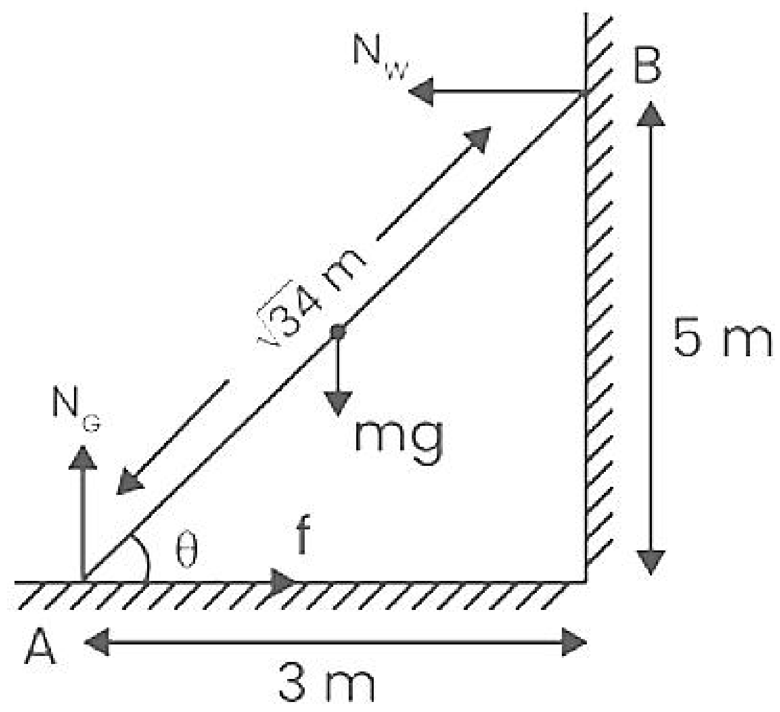
Taking moments about bottom point of ladder

$$N_w \times 5 = mg \times \frac{\sqrt{34}}{2}\cos\theta$$

$$N_w \times 5 = 100 \times \frac{\sqrt{34}}{2} \times \frac{3}{\sqrt{34}}$$

$$N_w = 30N$$

$$\frac{N_w}{N_G} = \frac{30}{100} = \frac{3}{10}$$



Question: If all the oxygen molecules dissociate into atoms and temperature is doubled then V_{rms} becomes____ times the original

Options:

(a) 4

(b) 3

(c) 2

(d) None of these

Answer: (c)

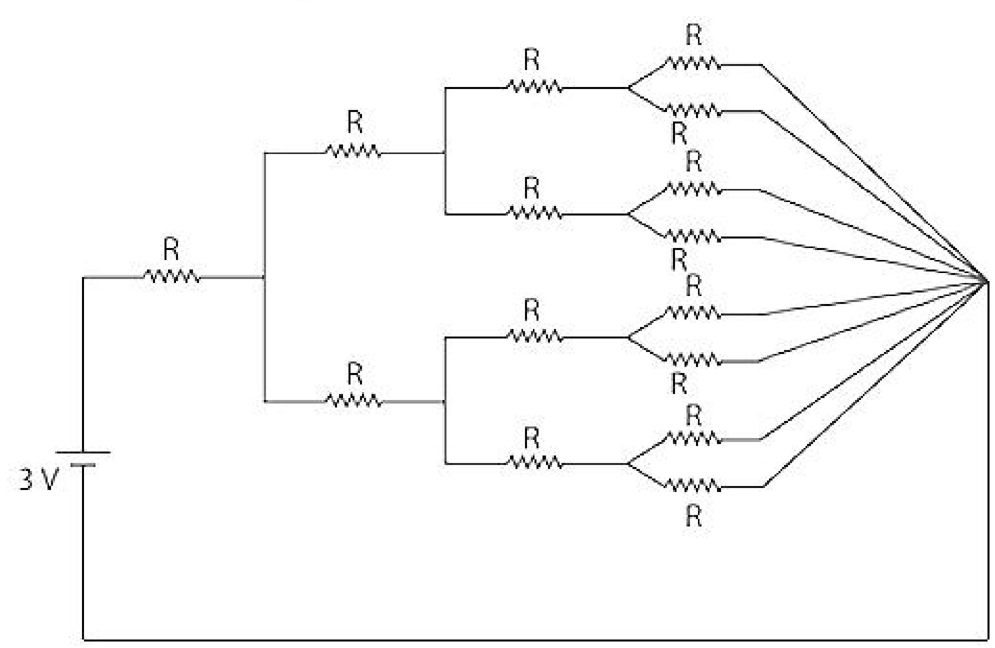
Solution:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\left(V_{rms}\right)_0 = \sqrt{\frac{3R(2T)}{M/2}}$$

$$= 2\sqrt{\frac{3RT}{M}} = 2V_{rms}$$

Question: $I = \frac{a}{5}, R = 1\Omega$, Find a.



Options:

(a) 14
(b) 8
(c) 12
(d) 2
Answer: (b)
Solution:
Starting from right most resistors
$R \| R$

$R_A = \frac{R}{2}\Omega$

$R + \frac{R}{2} = \frac{3R}{2}\Omega$

$\frac{3R}{2} \| \frac{3R}{2}$

$R_B = \frac{3R}{4}\Omega$

$R + \frac{3R}{4} = \frac{7R}{4}\Omega$

$\frac{7R}{4} \| \frac{7R}{4}$

$R_C = \frac{7R}{8}\Omega$

$\mathrm{R}_{eq} = R + \frac{7R}{8} = \frac{15R}{8}\Omega = \frac{15}{8}\Omega$

$I = \frac{V}{R} = \frac{3}{15} \times 8 = \frac{8}{5}A$

$\Rightarrow a = 8$

Question: In YDSE slab of thickness t and RI 1.5 is inserted in front of one of the slits. As a result intensity at the central maxima remains the same. What is the minimum value of thickness required?
Options:
(a) 2λ
(b) 4λ
(c) 8λ
(d) None of these
Answer: (a)
Solution:

Shift $= \frac{D}{d}(\mu - 1)t$

$= \frac{D\lambda}{d}$

$\Rightarrow t = \frac{\lambda}{\mu - 1} = 2\lambda$

Question: If resistance of a resistor is 2 Ω at 10°C and it is 3 Ω at 30°C find the temperature coefficient of resistance
Options:
(a) $0.24 \times 10^{-2} / °C$

(b) $4.4\times10^{-2}\ /\ ^{\circ}C$
(c) $2.5\times10^{-2}\ /\ ^{\circ}C$
(d) None of these
Answer: (c)
Solution:

$$R_t = R_0\left(1+\alpha\Delta T\right)$$
$$3 = 2\left(1+\alpha\left(30-10\right)\right)$$
$$3 = 2+40\alpha$$
$$1 = 40\alpha$$
$$\alpha = \frac{1}{40} = 2.5\times10^{-2}\ /\ ^{\circ}C$$

Question: Particle moves along the straight line such that it moves 1/3rd distance with speed v_1 the next 1/3rd distance with speed v_2 and remaining 1/3rd distance with speed v_3. Then its average speed throughout motion is
Options:
(a) $\dfrac{v_1v_2+v_2v_3+v_3v_1}{v_1+v_2+v_3}$
(b) $\dfrac{v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$
(c) $\dfrac{v_1+v_2+v_3}{3}$
(d) $\dfrac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_3v_1}$
Answer: (d)
Solution:
Average speed = total distance covered / total time taken
Let the total distance = 3x

Time taken to cover first one third $\left(x\right)=t_1=\dfrac{x}{v_1}$

Time taken to cover second one third $\left(x\right)=t_2=\dfrac{x}{v_2}$

Time taken to cover third one third $\left(x\right)=t_3=\dfrac{x}{v_3}$

Average speed $=\dfrac{3x}{\dfrac{x}{v_1}+\dfrac{x}{v_2}+\dfrac{x}{v_3}}$

$$=\frac{3x}{x\left(\dfrac{v_3v_2+v_1v_3+v_1v_2}{v_1v_2v_3}\right)}$$

$$=\frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_1v_3}$$

Question: A water drop of radius 1 μm in falls through air. Force of buoyancy and density of air is negligible. If the coefficient of viscosity of air is $2.0\times10^{-5} kgm^{-1}s^{-1}$. Find terminal velocity of water drop.

Options:

(a) $3.4\times10^{-4} m/J$

(b) $2.4\times10^{-4} m/J$

(c) $1.4\times10^{-4} m/J$

(d) $1.1\times10^{-4} m/J$

Answer: (d)

Solution:

$$V_T = \frac{2}{9}\frac{r^2(\gamma-6)g}{\eta}$$

$$V_T = \frac{2}{9}\frac{r^2\gamma g}{\eta}$$

$$V_T = \frac{2}{9}\times\frac{(1\times10^{-6})^2\times1\times10^3\times10}{2.0\times10^{-5}}$$

$$V_T = \frac{1}{9}\times10^{-12+3+1+5} = 1.1\times10^{-4} m/J$$

Question: Two capacitors of capacities 5μF and 10μF connected and the switch is kept open. Initially potential on 5μF capacitor is 30V and 10μF capacitor is uncharged. Find the charge on the 10μF capacitor once the switch is closed.

Options:

(a) 300 μC

(b) 100 μC

(c) 200 μC

(d) 400 μC

Answer: (b)

Solution:

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V = \frac{5\times30}{5+10} = 10V$$

$$Q = 10\times10 = 100\mu C$$

Question: Two bodies of equal mass has force of attraction F, then find the the force of attraction when one third of mass of one body is transferred to another

Options:

(a) $\frac{1}{9}F$

(b) $\frac{8}{9}F$

(c) $\frac{5}{9}F$

(d) $\frac{7}{9}F$

Answer: (b)
Solution:

$$F_i = \frac{Gm^2}{r^2}$$

$$F_f = \frac{G\left(m - \frac{1}{3}m\right)\left(m + \frac{1}{3}m\right)}{r^2}$$

$$= G\frac{\frac{8}{9}m^2}{r^2} = \frac{8}{9}F_i = \frac{8}{9}F$$

Question: A coil is placed in a time varying magnetic field. If the no. of turns are halved and the radius of wire is doubled. (Assume the coil to be short circuited) Then the power dissipated:

Options:

(a) $4P_i$

(b) $1P_i$

(c) $7P_i$

(d) $3P_i$

Answer: (a)
Solution:

$N_i = n \quad N_f = \frac{n}{2}$

$r_i = r \quad r_f = 2r$

Total length of wire $= n(2\pi R) =$ where R is radius of loop finally if n becomes half, radius of loop has to double

$\therefore$ New $emf = -\left(\frac{n}{2}\right)\left(\pi(2R)^2\right)\frac{dB}{dt}$

New Power $= \dfrac{\left[-\frac{n}{2}\pi\left(4R^2\right)\frac{dB}{dt}\right]^2}{\rho\frac{l}{\pi(2r)^2}}$

Old power$= \dfrac{\left[-n\pi(R)^2(dB/dt)\right]^2}{\rho\frac{l}{\pi r^2}}$ $\therefore P_f = 4P_i$

Question: K_1 and K_2 are KE_{max} of λ_1 and λ_2 Falling a metal If $\lambda_1 = 3\lambda_2$ Find relation of K_1 and K_2

Options:

(a) $3K_1 < K_2$

(b) $4K_1 < K_2$

(c) $5K_1 < K_2$

(d) $2K_1 < K_2$

Answer: (a)

Solution:

Kinetic energy of the photoelectrons $K = \frac{hc}{\lambda} - \phi$ where ϕ is the work function of the metal

$\therefore$ For wavelength λ_1 $K_1 = \frac{hc}{\lambda_1} - \phi ...(1)$

For wavelength λ_2, $K_2 = \frac{hc}{\lambda_2} - \phi ...(2)$

Given: $\lambda_1 = 3\lambda_2$

$\therefore$ Equation (1) becomes $K_1 = \frac{hc}{3\lambda_2} - \phi ...(3)$

From (2) - (3), we get $K_2 - K_1 = \frac{hc}{\lambda_2} - \frac{hc}{3\lambda_2}$

$K_2 - K_1 = \frac{2}{3}\frac{hc}{\lambda_2} \quad \Rightarrow \frac{hc}{\lambda_2} = \frac{3}{2}(K_2 - K_1)$

Put this in (2), $K_2 = \frac{3}{2}(K_2 - K_1) - \phi$

$\Rightarrow K_2 - 3K_1 = 2\phi$

As $\phi > 0 \Rightarrow K_2 - 3K_1 > 0$

Thus $K_1 < \frac{K_2}{3}$

Question: EM wave is moving in +x direction. If amplitude of electric field is $E_0 = 60\,N/C$ which is oscillating in y direction, then find the equations of E and B

Options:

(a) $E = 60\sin(kx - \omega t)\hat{i}$
$B = 2\times10^{-7}\sin(k_x - \omega t)\hat{j}$

(b) $E = 60\sin(kx - \omega t)\hat{k}$
$B = 2\times10^{-7}\sin(k_x - \omega t)\hat{j}$

(c) $E = 60\sin(kx - \omega t)\hat{j}$
$B = 2\times10^{-7}\sin(k_x - \omega t)\hat{k}$

(d) $E = 60\sin(kx - \omega t)\hat{k}$
$B = 2\times10^{-7}\sin(k_x - \omega t)\hat{i}$

Answer: (c)

Solution:

$E_0 = 60$

$\therefore B_0 = \frac{E_0}{C} = \frac{60}{3\times10^8} = 2\times10^{-7}$

Since wave is moving in +x-dir

$E = 60\sin(kx - \omega t)\hat{j}$
$B = 2\times10^{-7}\sin\left(k_x - \omega t\right)\hat{k}$

Question: A solenoid is filled with material of susceptibility 2×10^{-7} Fractional change in field intensity compared to the case when air was present inside instead of material

Options:

(a) $= 2\times10^{-5}\%$
(b) $= 4\times10^{-5}\%$
(c) $= 3\times10^{-5}\%$
(d) $= 5\times10^{-5}\%$
Answer: (a)
Solution:

$$X_m = \frac{M}{H}$$

It is already fractional change in the magnetic induction due to the medium.

$\therefore$ % age change

$= X_m \times 100$
$= 2\times10^{-7}\times100$
$= 2\times10^{-5}\%$

Question: Half life of radioactive material is 200 days. Find percent of substance remaining in 83 days
Options:

(a) 65%
(b) 55%
(c) 44%
(d) 75%
Answer: (d)
Solution:

$$N = N_0\left(\frac{1}{2}\right)^{t/T}$$

$T =$ half life
$t =$ time elapsed

$$\therefore N = N_0\left(\frac{1}{2}\right)^{\frac{83}{200}}$$

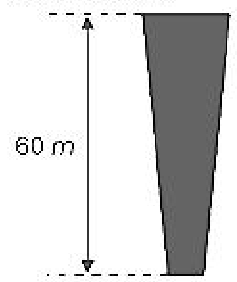
$$\frac{N}{N_0}\times100 = \left(\frac{1}{2}\right)^{\frac{83}{200}}\times100$$

$\approx 75\%$

Question: Water falls at a rate pf 600 kg/s from a height of 60 m as shown. How many bulbs of capacity 100 W each will glow from the energy produced at the bottom of the fall. Assume full conversion of energy of falling water and all bulbs glowing at 100 W each.

Options:

(a) 25
(b) 50
(c) 3600
(d) 1000
Answer: (c)
Solution:



Potential energy loss per second

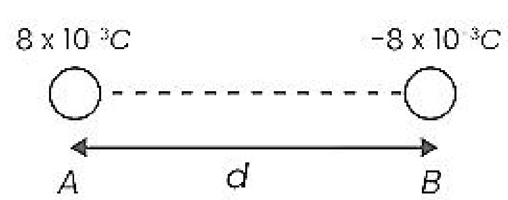
$= 600(10)(60) \mathrm{J/s}$

$= 36 \times 10^4 \mathrm{J/s} = 36 \times 10^4 \mathrm{W}$

Each bulb consumes 100 W,

$\therefore$ Total no of bulbs which can glow is 3600.

Question: Two opposite charges are placed at a distance d as shown. Electric filed strength at mid point is $6.4 \times 10^4 \mathrm{N/C}$. Then the value of d is



Options:

(a) $20\sqrt{10}$
(b) $50\sqrt{10}$
(c) $30\sqrt{10}$
(d) $60\sqrt{10}$
Answer: (c)
Solution:

$$E = \frac{2kQ}{(d/2)^2}$$

$$6.4 \times 10^4 = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-3}}{d^2/4}$$

$$\therefore d^2 = 9000$$

$$d = 30\sqrt{10}$$

Question: In series RLC circuit voltage across capacitance and inductance is twice that of resistance. If $R = 50\text{hm}, V = 220\,\text{V}, \text{f} = 50\,\text{Hz}$. If $L = 1/k\pi$ then value of k is (in m H)

Options:

(a) 11^{-2}

(b) 10^{-2}

(c) 12^{-2}

(d) 15^{-2}

Answer: (b)

Solution:

$$V_L = 2V_R$$

$$\omega L = 2R$$

$$L = \frac{2R}{\omega} = \frac{2R}{2\pi f} = \frac{2(s)}{2\pi(50)}$$

$$= \frac{1}{10\pi} = \frac{1}{10\pi} \times \frac{10^{-3}}{10^{-3}}$$

$$\therefore K = 10^{-2}$$

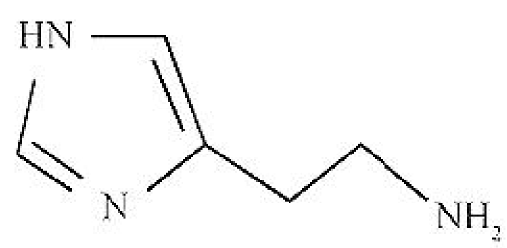
JEE-Main-28-06-2022-Shift-2 (Memory Based)

Chemistry

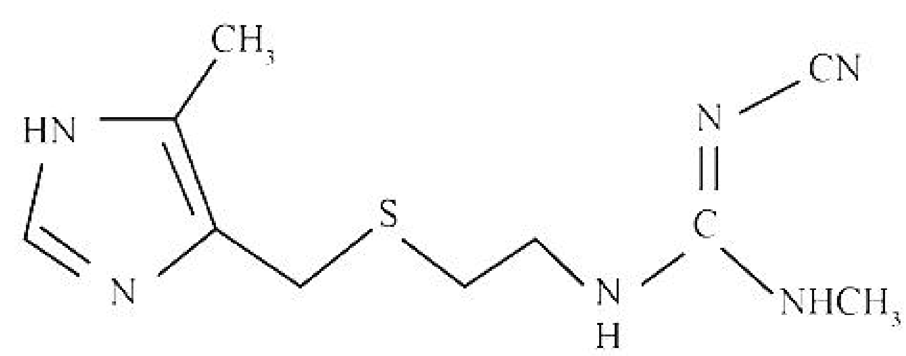
Question: Which of the following is the structure of Tagamet?

Options:

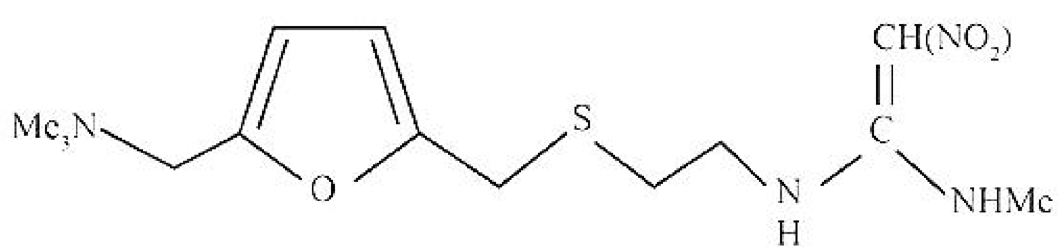
(a)



(b)



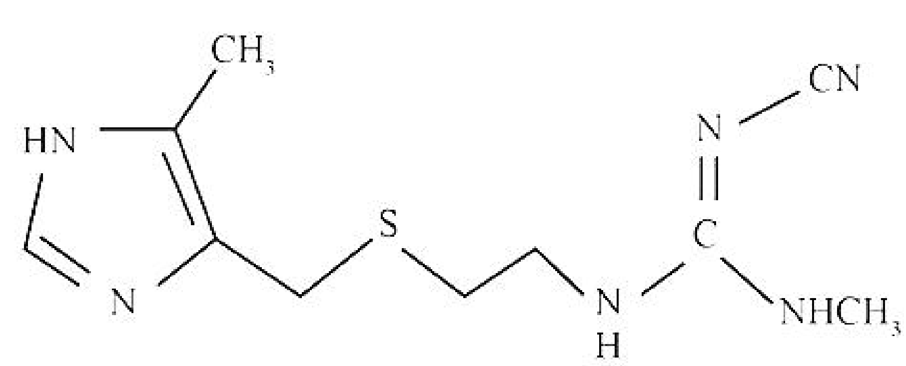
(c)



(d) None of these

Answer: (b)

Solution:



Question: In which of oxyacids of sulphur both sulphur have different oxidation state.

Options:

(a) $H_2S_4O_6$

(b) $H_2S_2O_8$

(c) $H_2S_2O_7$

(d) All of these

Answer: (a)

Solution:

$$\begin{array}{ccccccccccc} & & O & & O & & O & & O & & \\ & & \| & & \| & & \| & & \| & & \\ OH & - & S & - & S & - & S & - & S & - & OH \\ & & +3 & & +2 & & +2 & & +3 & & \end{array}$$

Question: Match the following.

Column-I	**Column-II**
(i) Positively charged	(A) Gel
(ii) Negatively charged	(B) Starch
(iii) Macromolecular starch	(C) CuS
(iv) Cheese	(D) $Fe_2CO_3 .x H_2O$

Options:

(a) (i) → (D); (ii) → (C); (iii) → (B); (iv) → (A)

(b) (i) → (B); (ii) → (C); (iii) → (A); (iv) → (D)

(c) (i) → (C); (ii) → (B); (iii) → (D); (iv) → (A)

(d) (i) → (D); (ii) → (A); (iii) → (D); (iv) → (B)

Answer: (a)

Solution:

(i) Positively charged ⇒ $Fe_2CO_3 .x H_2O$

(ii) Negatively charged ⇒ CuS

(iii) Macromolecular starch ⇒ Starch

(iv) Cheese ⇒ Gel

Question: A compound has 8% H, 70%C, 16% N, Molecular Mass is 160. Find the formula of compound.

Options:

(a) $C_{12}H_{16}N_2$

(b) $C_{12}H_{18}N_2$

(c) $C_{11}H_{16}N$

(d) $C_{12}H_{15}N$

Answer: (a)

Solution:

Compound contain 8% H, 70% C and 16% N

No. of moles of C = $\frac{70}{12} = 5.8 \approx 6$

No. of moles of H = $\frac{8}{1} = 8$

No. of moles of N = $\frac{16}{14} = 1 : 1 \approx 1$

Mole ratio C : H : N = 6 : 8 : 1

Empirical formula = C_6H_8N

Molecular mass = 160

Empirical formula mass = $12 \times 6 + 2 \times 1 + 14 = 94$

n = 2

Formula of compound = $(C_6H_8N)_2 = C_{12}H_{16}N_2$

Question: What is correct about photochemical smog?

Options:

(a) It is reducing in nature

(b) It occurs in humid conditions

(c) It is formed due to the action of sunlight on Hydrocarbons

(d) All of these

Answer: (c)

Solution: Photochemical smog results from the action of sunlight on hydrocarbons.

Question: Which of the following is basic oxide?

Options:

(a) CaO

(b) SiO_2

(c) Al_2O_3

(d) NO

Answer: (a)

Solution: CaO - basic oxide

SiO_2 - acidic oxide,

Al_2O_3 - Amphoteric oxide,

NO - neutral oxide

Question: An ideal gas is stored in a vessel of volume 416 ml at temperature 300 K and Pressure 1.5 atm. What is the mass of gas? (Molecular mass of gas 100g/mol)

Options:

(a) 3.32 g

(b) 2.53 g

(c) 3.01 g

(d) 1.92 g

Answer: (b)

Solution:

$$PV = \frac{w}{M} RT$$

$$\therefore w = \frac{PVM}{RT} = \frac{1.5 \times 0.416 \times 100}{0.0821 \times 300} = 2.53\,g$$

Question: 2.5 g of protein taken and made 500 ml of solution. Osmotic pressure of solution is 5.03×10^{-3} at 300 K. Find the no. of glycine unit.

Options:

(a) 1.9×10^{16} units

(b) 2.8×10^{15} units

(c) 1.2×10^{15} units

(d) 2.2×10^{10} units

Answer: (c)

Solution:

Osmotic pressure $(\pi) = \left(\frac{n_2}{V}\right)RT$

$$\pi V = \frac{w_2 RT}{M_2}$$

$$M_2 = \frac{w_2 RT}{\pi} = \frac{2.5 \times 0.0821 \times 300}{5.03 \times 10^{-8}} = 12.2 \times 10^{8}\ g$$

No. of glycine units $= \frac{2.5 \times 6.023 \times 10^{23}}{12.2 \times 10^{8}} = 1.2 \times 10^{15}$ units

Question: In SF_4 what is the bond angle?

Options:

(a) 90°, 120°

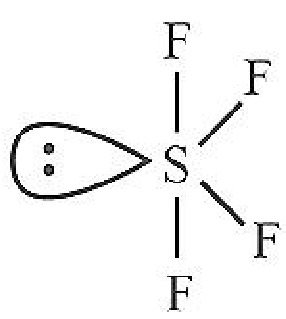
(b) 90°, 117°

(c) 89°, 120°

(d) 89°, 117°

Answer: (d)

Solution:



Question: The volume of 0.01 M $KMnO_4$ solution which can oxidize 20 ml of 0.05 M Mohr salt solution in acidic medium is

Options:

(a) 10 ml

(b) 20 ml

(c) 30 ml

(d) 40 ml

Answer: (b)

Solution: $M_1V_1Z_1 = M_2V_2Z_2$

$0.01 \times V_1 \times 5 = 0.05 \times 20 \times 1$

$V_1 = 20$ ml

Question: In the buffer solution, having pH = 4 and $Pk_a = 1.3 \times 10^{-5}$, find the ratio of salt/acid is

Options:

(a) $10^{-0.8}$

(b) 0.1

(c) $10^{0.8}$

(d) $10^{-2.1}$

Answer: (a)

Solution: $pK_a = -\log(1.3 \times 10^{-3})$

$= 5 - \log 1.3$

$= 4.85$

$$pH = pKa + \log\frac{[\text{salt}]}{[\text{acid}]}$$

$$4 = 4.8 - \log\frac{[\text{salt}]}{[\text{acid}]}$$

$$-0.8 = \log\frac{[\text{salt}]}{[\text{acid}]}$$

$$\frac{\text{salt}}{\text{acid}} = 10^{-0.8}$$

Question: Nitration of aniline with $HNO_3 + H_2SO_4$ gives

Options:

(a) p-nitroaniline

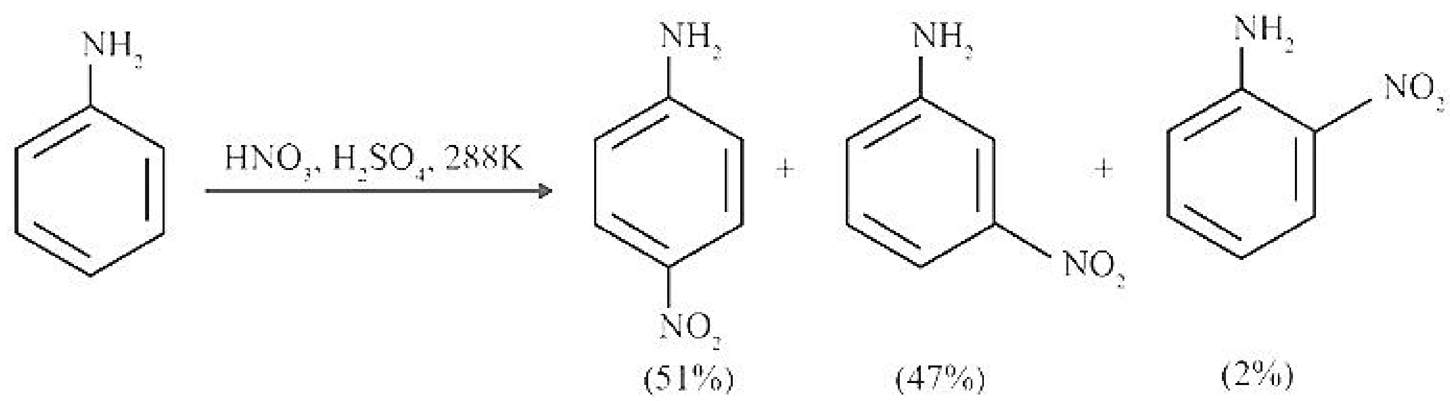
(b) m-nitroaniline

(c) o-nitroaniline

(d) All of these

Answer: (d)

Solution:



Question: Assertion: Natural form of rubber is cis 1,4 polyisoprene

Reason: There are weak vander waals forces giving its coiled structure

Options:

(a) Both assertion and reason are true, reason is correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not a correct explanation of assertion.

(c) Assertion is true, but reason is false

(d) Assertion is false, but reason is true

Answer: (b)

Solution: Both assertion and reason are true, but reason is not correct explanation of A

Natural rubber may be considered as a linear polymer of isoprene (2-methyl-1, 3-butadiene) and is also called as cis-1, 4-polyisoprene.

The cis-polyisoprene molecule consists of various chains held together by weak van der Waals interactions and has a coiled structure. Thus. It can be stretched like a spring and exhibits elastic properties.

Question: In extraction of copper FeO and $FeSiO_3$ are respectively

Options:

(a) slag, gangue

(b) gangue, slag

(c) both are slag

(d) both are gangue

Answer: (b)

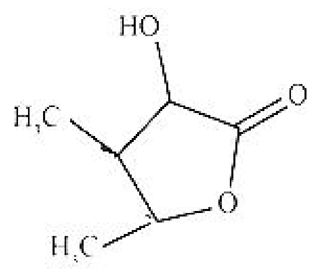
Solution: FeO is gangue and SiO_2 is flux to form slag $FeSiO_3$.

$$\underset{\text{basic impurity}}{FeO} + \underset{\text{acidic flux}}{SiO_2} \rightarrow \underset{\text{slag}}{FeSiO_3}$$

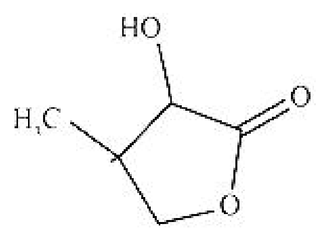
Question: Isobutyraldehyde reacts with K_2CO_3 and formaldehyde to give A. A reacts with HCN to give B. Hydrolysis of B gives a stable carboxylic acid C. What is C?

Options:

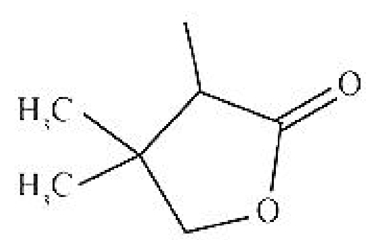
(a)



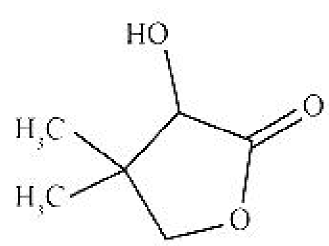
(b)



(c)

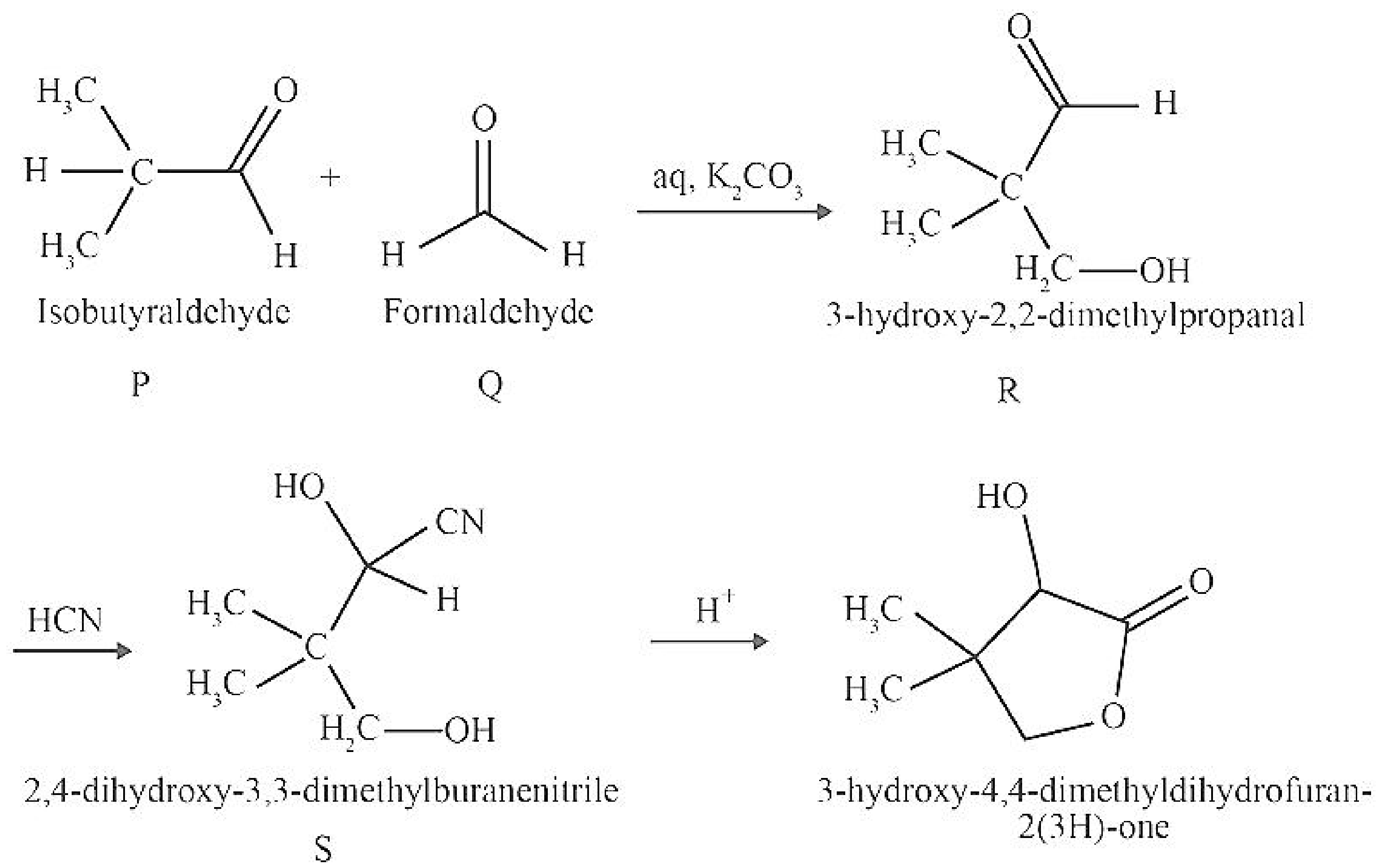


(d)



Answer: (d)

Solution:



Question: The isotopes of Hydrogen differ in the following property

Options:

(a) Electronic configuration

(b) No of protons

(c) Atomic number

(d) Atomic mass

Answer: (d)

Solution: The three isotopes of hydrogen differ in mass numbers which are 1, 2 and 3 respectively known as protium, deuterium and tritium.

Question: X reacts with Br_2/H_2O to give gluconic acid and reacts with HNO_3 to give saccharic acid. Name X

Options:

(a) Maltose

(b) Starch

(c) Fructose

(d) Glucose

Answer: (d)

Solution:

$$\begin{array}{c} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \xrightarrow{Br_2 \text{ water}} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \\ \text{Gluconic acid} \end{array}$$

$$\begin{array}{c} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \xrightarrow{\text{Oxidation}} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \\ \text{Saccharic acid} \end{array}$$

Question: Comparison of KE for wavelengths λ and 3 λ, keeping work function constant

Options:

(a) $K.E_2 = 9\ K.E_1$

(b) $K.E_2 = \frac{1}{9}\ K.E_1$

(c) $K.E_2 = 3\ K.E_1$

(d) $K.E_2 = \frac{1}{3}\ K.E_1$

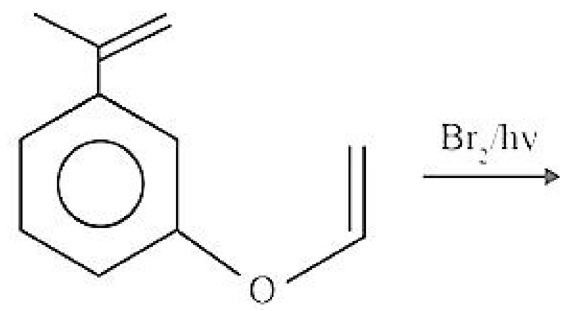
Answer: (b)

Solution: $\lambda = \frac{h}{\sqrt{2m\,K.E}}$

$$\frac{\lambda}{3\lambda} = \frac{\frac{h}{\sqrt{2m\,K.E_1}}}{\frac{h}{\sqrt{2m\,K.E_2}}}$$

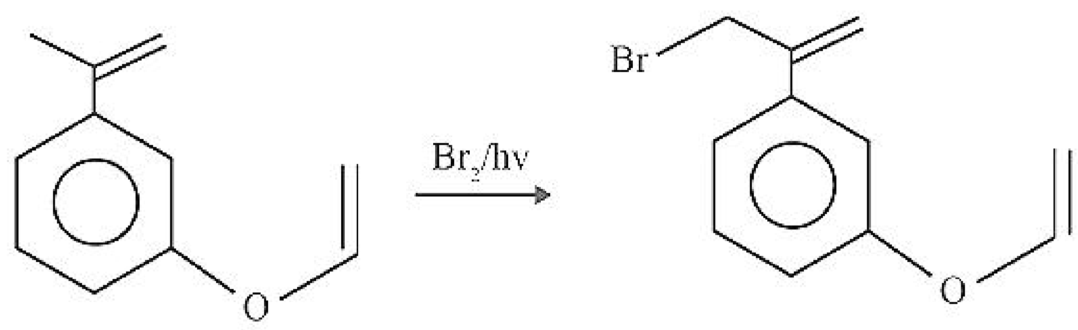
$$K.E_2 = \frac{1}{9} K.E_1$$

Question: Product will have how many Br……..



Answer: 1.00

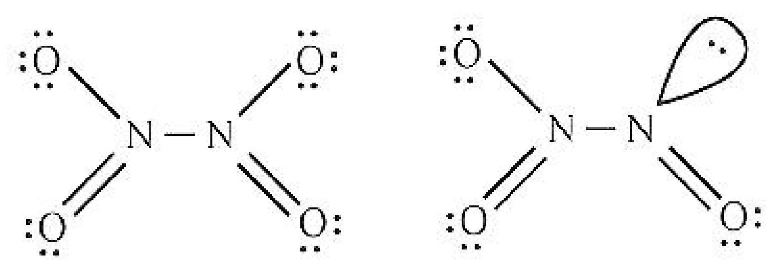
Solution:



Question: How many of the following contain N – N bond N_2O, N_2O_3, N_2O_4, N_2O_5?

Answer: 2.00

Solution: N_2O_4 and N_2O_3 has one N – N bond as shown below



Question: Consider the following complexes $[Fe(CN)_6]^{3-}$, $[Ni(CN)_4]^{2-}$ and $[Fe(CN)_6]^{4-}$. How many complex(es) is/are paramagnetic?

Answer: 1.00

Solution:

$[Fe(CN)_6]^{3-} \rightarrow Fe^{3+} \rightarrow 3d^5 \rightarrow$ paramagnetic

$[Ni(CN)_4]^{2-} \rightarrow Ni^{2+} \rightarrow 3d^8 \rightarrow$ diamagnetic

$[Fe(CN)_6]^{4-} \rightarrow Fe^{2+} \rightarrow 3d^6 \rightarrow$ diamagnetic

JEE-Main-28-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: If n arithmetic means are inserted between a and 100 then ratio of first AM and n^{th} AM is $1:7$ and $a+n=33$. Find n.

Options:

(a) 21

(b) 22

(c) 23

(d) 24

Answer: (c)

Solution:

$$d=\frac{100-a}{n+1}$$

$$A_1=a+\frac{100-a}{n+1}$$

$$A_n=a+n\left(\frac{100-a}{n+1}\right)$$

$$\frac{A_1}{A_n}=\frac{an+100}{a+100n}=\frac{1}{7}$$

$$\Rightarrow 7an+700=a+100n$$

$$a+n=33$$

$$\Rightarrow a=33-n$$

$$\Rightarrow 7(33-n)n+700=33-n+100n$$

$$\Rightarrow 231n-7n^2+700=33+99n$$

$$\Rightarrow 7n^2-132n-667=0$$

$$\Rightarrow n=23,\ \frac{-29}{7}$$

$$23\times\alpha=\frac{-667}{7}$$

$$\Rightarrow n=23$$

Question: Find the area enclosed by x-axis & $y=3-|x+1|-\left|x-\frac{1}{2}\right|$.

Options:

(a) $\frac{27}{8}$

(b) $\frac{23}{8}$

(c) $\frac{25}{8}$

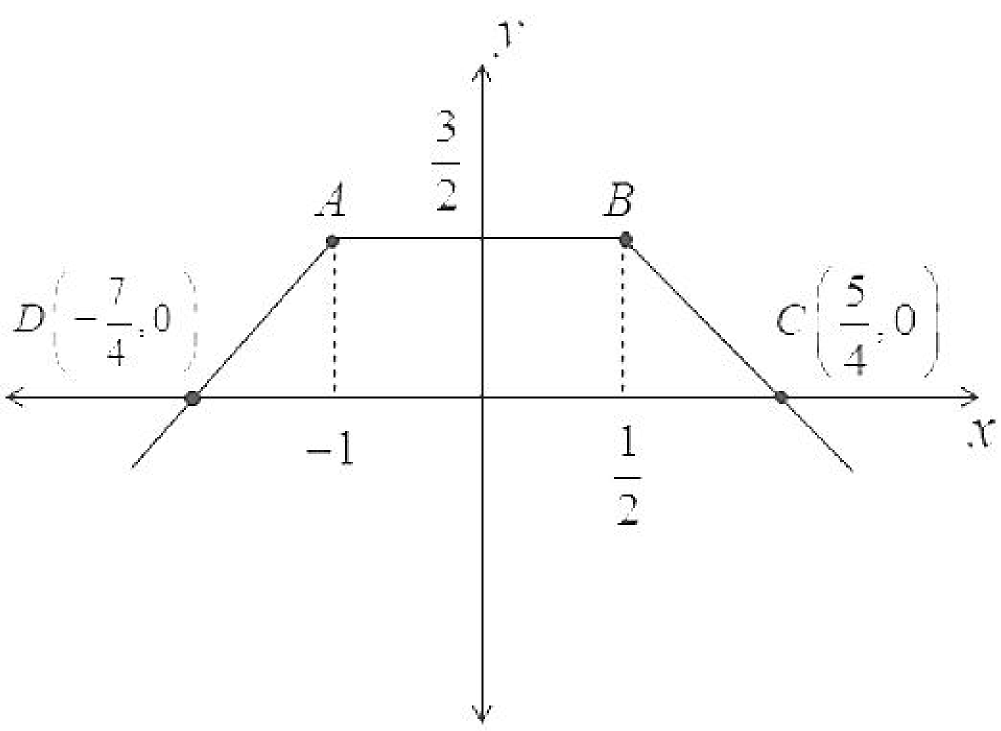
(d) $\frac{27}{4}$

Answer: (d)

Solution:

$$\begin{cases} 3+x+1+x-\frac{1}{2} & ; \quad x<-1 \\ 3-x-1+x-\frac{1}{2} & ; \quad -1 \le x < \frac{1}{2} \\ 3-x-1-x+\frac{1}{2} & ; \quad x \ge \frac{1}{2} \end{cases}$$

$$\begin{cases} 2x+\frac{7}{2} & ; \quad x<-1 \\ \frac{3}{2} & ; \quad -1 \le x < \frac{1}{2} \\ -2x+\frac{5}{2} & ; \quad x \ge \frac{1}{2} \end{cases}$$



Required area $(ABCD) = \frac{1}{2} \times \left(3+\frac{3}{2}\right)\frac{3}{2}$

$= \frac{27}{8}$

Question: $f(x)+f(x+k)=n,\ I_1=\int_0^{4nk} f(x)\,dx,\ I_2=\int_{-k}^{3k} f(x+k)\,dx$

Options:

(a) $I_1 = 2I_2$

(b) $I_1 = nI_2$

(c)

(d)

Answer: (b)

Solution:

$f(x)+f(x+k)=n$

$x \to x+k$

$f(x+k)=f(x+2k)=n$

$\Rightarrow f(x)=f(x+2k)$

Period $=2k$

$$I_1 = \int_0^{4nk} f(x)dx = 2n\int_0^{2k} f(x)dx$$

$$I_2 = \int_{-k}^{3k} f(x+k)dx = \int_0^{4k} f(t)dt$$

$$=2\int_0^{2k} f(x)dx$$

$I_1 = nI_2$

Question: There are 30 candidates to be distributed among 4 children C_1, C_2, C_3, C_4 such that C_2 gets at least 4 and at most 7 candy and C_3 gets atleast 2 and almost 6 candy. The number of ways to distribute it.

Answer: ${}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

Solution:

$x_1 + x_2 + x_3 + x_4 = 30$

$4 \le x_2 \le 7$

$t_2 = x_2 - 4$

$2 \le x_3 \le 6$

$t_3 = x_3 - 2$

$\Rightarrow x_1 + t_2 + t_3 + x_4 = 24$

${}^{24+4-1}C_{4-1} = {}^{27}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$8 \le 2,\ \ 2 \le x_3$

$x_1 + t_2 + t_3 + x_4 = 20$

$^{20+4-1}C_{4-1} = {}^{23}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$4 \le x_2,\ 7 \le x_3$

$x_1 + t_2 + t_3 + x_4 = 19$

$^{19+4-1}C_{4-1} = {}^{22}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$8 \le x_2,\ 7 \le x_3$

$x_1 + t_1 + t_2 + x_4 = 15$

$^{15+4-1}C_{4-1} = {}^{18}C_3$

Required answer $= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

Question: $f(x)$ is a quadratic polynomial. If $f(-2)+f(3)=0$ and one root of equation is -1. Find the sum of roots.

Answer: $\frac{11}{3}$

Solution:

$f(x) = ax^2 + bx + c$

$f(-1) = 0 \Rightarrow a - b + c \qquad ....(1)$

$f(-2) + f(3) = 0$

$\Rightarrow (4a - 2b + c) + (9a + 3b + c) = 0$

$13a + b + 2c = 0 \qquad ....(2)$

Eq. (2) – 2 (Eq. 1)

$\Rightarrow 11a + 3b = 0$

$\Rightarrow b = \frac{-11a}{3}$

Sum of roots $= \frac{-b}{a} = \frac{11}{3}$

Question: $\lim\limits_{n \to \infty} 6 \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right)$

Answer: 3.00

Solution:

Given, $\lim\limits_{n \to \infty} 6 \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right)$

$\lim\limits_{n \to \infty} 6 \tan\left(\sum_{r=1}^{n} \tan^{-1}\left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)}\right]\right)$

$$= \lim_{n\to\infty} 6\tan\left(\sum_{r=1}^{n} \tan^{-1}(r+2) - \tan^{-1}(r+1)\right)$$

$$= \lim_{n\to\infty} 6\tan\left[\tan^{-1}(n+2) - \tan^{-1}(2)\right]$$

$$= \lim_{n\to\infty} 6\tan\left[\tan^{-1}\left(\frac{n}{1+2(n+2)}\right)\right]$$

$$= \lim_{n\to\infty} \frac{6n}{2n+5} = 3$$

Question: If $\lim_{x\to 1}\left(\frac{\sin(3x^2-4x+1)-4x+1}{2x^3-7x^2+ax+b}\right) = -2$ then $a-b=?$

Answer: 11.00

Solution:

$$\lim_{x\to 1}\left[\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}\right] = -2$$

$$\Rightarrow a+b=5$$

$$\lim_{x\to 1}\frac{\cos(3x^2-4x+1)(6x-4)-2x}{6x^2-14x+a} = -2$$

$$\Rightarrow a=8,\ b=-3$$

$$\therefore\ a-b=11$$

Question: For a parabola directrix: $3x-4y=21$, vertex (2, -1). Find length of Latus Rectum.

Answer: $\frac{32}{5}$

Solution:

Directrix: $3x-4y=21$; vertex (2, -1)

$$a = \left|\frac{3(2)-4(-1)-21}{5}\right| = \left|\frac{6+4-21}{5}\right| = \left|\frac{11}{5}\right|$$

$$\therefore\ \text{LLR} = 4a = \frac{44}{5}$$

Question: $\cot\alpha = 1$; $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$, $\sec\beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$. Find $\tan(\alpha+\beta)$.

Answer: $\frac{-1}{7}$

Solution:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1-\frac{4}{3}}{1+\frac{4}{3}}=\frac{-1}{7}$$

Question: $A=\{a,b,c,d\}, B=\{1,2,3,4,5\}, f:A\to B$ is one-one. Find the probability that $f(a)+2f(b)-f(c)=f(d)$.

Answer: $\frac{1}{20}$

Solution:

$$f(b)=\frac{f(c)+f(d)-f(a)}{2}$$

a	b	c	d
1	3	2	5
5	1	3	4
4	2	3	5
5	1	4	3
1	3	5	2
4	2	5	3

Favourable cases = 6

Total cases $=5\times4\times3\times2$

$\therefore$ Required probability $=\frac{6}{5!}=\frac{1}{20}$

Question: Find the coefficient of term independent of x in $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$.

Answer: $\frac{33}{200}$

Solution:

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is:

$$T_{r+1}={}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$$

$$={}^{11}C_r(-1)^r\cdot\frac{5^{11-2r}}{2^{11-r}}\cdot x^{33-5r}$$

Coefficient of term independent of $x=$ Coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

-Coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ $+3\times$ Coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$$=0-{}^{11}C_7(-1)^7\cdot\frac{5^{-3}}{2^4}+0=\frac{330}{5^3\cdot 2^4}=\frac{33}{200}$$

Question: If vertex of parabola is (2, -1) and equation of its directrix is $4x-3y=21$, then the length of latus rectum is

Answer: 8.00

Solution:

Length if Latus rectum = 4 (perpendicular distance of (2, -1) from $4x-3y-21=0$)

$$=4\frac{|8+3-21|}{5}=8$$

Question: Find the equation of plane passing through the points $(2,-1,0)$ and perpendicular to planes $2x-3y+z=0$ and $2x-y-3z=0$.

Answer: 6.00

Solution:

Let DRs be a,b,c

Now $2a-3b+c=0$

$2a-b-3c=0$

$\frac{a}{5}=\frac{b}{4}=\frac{c}{2}$

Equation of required plane is

$5(x-2)+4(y+1)+2(z-0)=0$

$\Rightarrow 5x+4y+2z=6$

Question: In an infinite GP, $a=n^2, r=\frac{1}{(n+1)^2}, \frac{1}{26}+\sum_{n=0}^{50}\left(S_n-\frac{2}{n+1}-n-1\right)=?$, where S_n is sum of given GP.

Answer: 41652.00

Solution:

$$a=n^2, r=\frac{1}{(n+1)^2};\ s_n=\frac{a}{1-r}=\frac{n(n+1)^2}{(n+2)}$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[\frac{n(n+1)^2}{(n+2)}+\left(\frac{2}{n+1}\right)-(n+1)\right]$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[\left(n^2-n-1\right)+\left(\frac{n}{n+2}\right)+\left(\frac{2}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[n^2-n-2\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\left[\frac{50\cdot 51\cdot 101}{6}-\frac{50\cdot 51}{2}-2\sum_{n=0}^{50}\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\left[41650-2\left(\frac{1}{52}-1\right)\right]=41652$$