JEE-Main-25-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $x^4+x^3+x^2+x+1=0$ equation has 4 roots $\alpha\beta\gamma\delta$. Then $\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=?$

Options:

(a) 4
(b) 1
(c) -1
(d) 4

Answer: (c)

Solution:

Given, $x^4+x^3+x^2+x+1=0$

$\Rightarrow x^5+x^4+x^3+x^2+x=0$

$\Rightarrow x^5-1=0$

$\Rightarrow x^5=1$

$\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}$

$=\alpha\left(\alpha^5\right)^{400}+\beta\left(\beta^5\right)^{400}+\gamma\left(\gamma^5\right)^{400}+\delta\left(\delta^5\right)^{400}$

$=\alpha+\beta+\gamma+\delta$

$=-1$

Question: A die is rolled twice and let the outcomes be α,β. Then probability such that $x^2+\alpha x+\beta>0$

Options:

(a) $\frac{17}{36}$
(b) $\frac{9}{36}$
(c)
(d)

Answer: (a)

Solution:

$x^2+\alpha x+\beta>0$

$\Rightarrow \alpha^2-4\beta<0$

$\Rightarrow \alpha=1,\ \beta=1,2,3,4,5,6$

$\Rightarrow \alpha=2,\ \beta=2,3,4,5,6$

$\Rightarrow \alpha=3,\ \beta=3,4,5,6$

$\Rightarrow \alpha=4,\ \beta=5,6$

$\Rightarrow \alpha=5,$ not possible

Favourable outcomes $=6+5+4+2=17$

$\therefore p=\frac{17}{36}$

Question: $\lim_{n\to\infty}\sqrt{n^2-n+1}+n\alpha+\beta=0$ then $8(\alpha+\beta)$

Options:
(a) -8
(b) -4
(c)
(d)
Answer: (b)
Solution:

$$\lim_{n\to\infty}\sqrt{n^2-n+1}+n\alpha+\beta=0$$

$$\lim_{n\to\infty}\left(\frac{\sqrt{n^2-n+1}+n\alpha+\beta}{\sqrt{n^2-n+1}-(n\alpha+\beta)}\right)\left(\sqrt{n^2-n+1}-(n\alpha+\beta)\right)=0$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^2-n+1-(n\alpha+\beta)^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$$

$$\Rightarrow \lim_{n\to\infty}\frac{n^2(1-\alpha^2)+n(-1-2\alpha\beta)+1-\beta^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$$

$\Rightarrow 1-\alpha^2=0$

$\Rightarrow \alpha=\pm 1$

$-1-2\alpha\beta=0$

$\beta=\frac{-1}{2\alpha}$

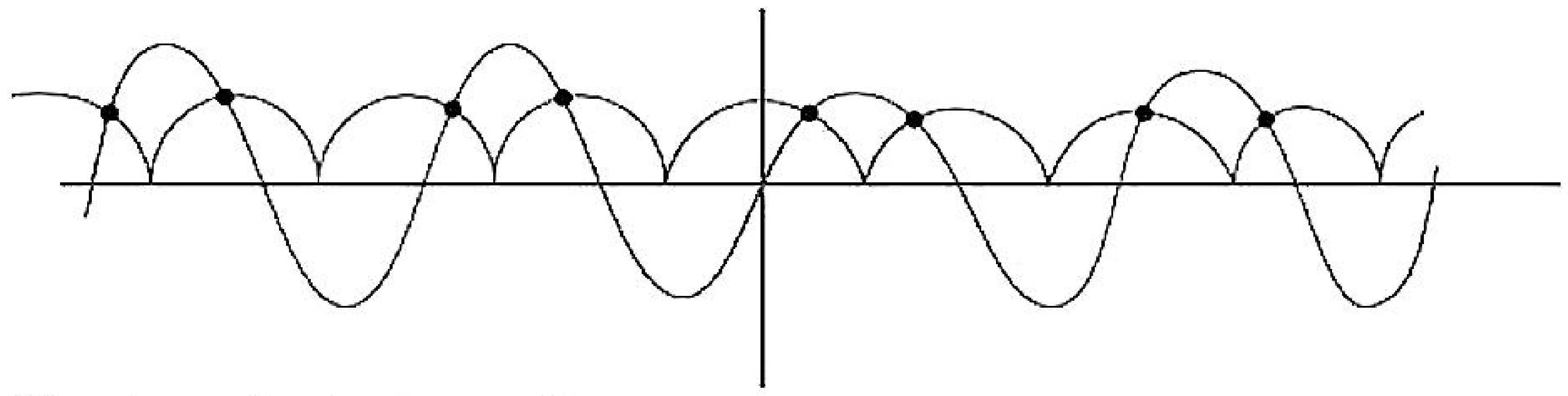
$\alpha=1, \beta=\frac{-1}{2}$

$\alpha=-1,\ \beta=\frac{1}{2}$

$8(\alpha+\beta)=\pm 4$

Question: The number of solution of $|\cos x|=\sin x$ such that $-4\pi\le x\le 4\pi$ is

Options:
(a) 4
(b) 6
(c) 8
(d) 12
Answer: (c)
Solution:



Number of solutions = 8

Question: Which of the following is a tautology?

Options:

(a) $(\sim p \vee q) \Rightarrow p$

(b) $p \Rightarrow (\sim p \vee q)$

(c) $(\sim p \vee q) \Rightarrow q$

(d) $q \Rightarrow (\sim p \vee q)$

Answer: (d)

Solution:

$q \Rightarrow (\sim p \vee q)$

$\sim q \vee (\sim p \vee q)$

$(\sim q \vee \sim p) \vee (\sim q \vee q)$

$(\sim q \vee \sim p) \vee (T)$

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Question: Set $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$. Find the number of functions from A to B such that $f(1) + f(2) = f(3)$

Answer: 40.00

Solution:

$A = \{1,2,3,4\}$, $B = \{1,2,3,4,5\}$

$f(1) + f(2) = f(3)$

$1 + 2 = 3 \rightarrow 2 \times 5 = 10$

$1 + 3 = 4 \rightarrow 2 \times 5 = 10$

$1 + 4 = 5 \rightarrow 2 \times 5 = 10$

$2 + 3 = 5 \rightarrow 2 \times 5 = 10$

Number of functions $= 10 + 10 + 10 + 10 = 40$

Question: If roots of $x^2 - 8ax + 2a = 0$ are $p \,\&\, r$, while $q \,\&\, s$ are roots of $x^2 + 12bx + 6b = 0$ then find $\frac{1}{a} - \frac{1}{b}$ if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ & $\frac{1}{s}$ are in AP.

Answer:

Solution:

$p + r = 8a,\ pr = 2a$

$q + s = -12b,\ qs = 6b$

$\Rightarrow \frac{1}{p}+\frac{1}{r}=4, \frac{1}{q}+\frac{1}{s}=-2$

$\Rightarrow \frac{1}{p}-\frac{1}{q}+\frac{1}{r}-\frac{1}{s}=6$

$\Rightarrow -2d=6$

$\Rightarrow d=-3$

$\frac{1}{p}+\frac{1}{p}-3=4$

$\Rightarrow \frac{2}{p}=7$

$\Rightarrow \frac{1}{p}=\frac{2}{7}, \frac{1}{q}=\frac{-19}{7}, \frac{1}{r}=\frac{-40}{7}, \frac{1}{s}=\frac{-61}{7}$

$\frac{1}{a}=\frac{2}{pr}=2\cdot\frac{2}{7}\cdot\left(\frac{-40}{7}\right)=\frac{-80}{49}$

$\frac{1}{b}=\frac{6}{qs}=6\cdot\left(\frac{-19}{7}\right)\left(\frac{-61}{7}\right)=\frac{6954}{49}$

$\frac{1}{a}-\frac{1}{b}=\frac{-80}{49}-\frac{6954}{49}$

$=\frac{-7034}{49}$

Question: If $a_1=b_1=1, a_n=a_{n-1}+2$ & $b_n=a_n+b_{n-1}$, then find $\sum_{n=1}^{15} a_n \times b_n$

Answer: 27560.00

Solution:

$a_1=b_1=1,\ a_n=a_{n-1}+2$

$a_r=1+(r-1)2=2r-1$

$b_r=a_r+b_{r-1}=a_r+a_{r-1}+b_{r-2}$

$=a_r+a_{r-1}+a_{r-2}+....+a_2+b_1$

$=\sum(2r-1)$

$=\frac{2(r)(r+1)}{2}-r=r^2$

$\sum a_n b_n=\sum(2n-1)n^2$

$=\sum 2n^3-\sum n^2$

$=2\left(\frac{15\times16}{2}\right)^2-\frac{15\times16\times31}{6}$

$=27560$

Question: A line with slope greater than 1, passes through $A(4,3)$. Line $x-y=2$ intersects former line at B. Find B if $AB=\frac{\sqrt{29}}{3}$.

Answer: $\frac{17}{3},\frac{11}{3}$

Solution:

$\frac{x-4}{\cos\theta}=\frac{y-3}{\sin\theta}=r$

$x=\frac{\sqrt{29}}{3}\cos\theta+4,\ y=\frac{\sqrt{29}}{3}\sin\theta+3$

$x-y=2$

$\Rightarrow\frac{\sqrt{29}}{3}\cos\theta+4-\frac{\sqrt{29}}{3}\sin\theta-3=2$

$\Rightarrow\frac{\sqrt{29}}{3}(\cos\theta-\sin\theta)=1$

$\Rightarrow\cos\theta-\sin\theta=\frac{3}{\sqrt{29}}$

$\Rightarrow 1-\sin 2\theta=\frac{9}{29}$

$\Rightarrow\sin 2\theta=\frac{20}{29}$

$\Rightarrow\sin\theta+\cos\theta=\sqrt{1+\left(\frac{20}{29}\right)}$

$\Rightarrow\sin\theta+\cos\theta=\frac{7}{\sqrt{29}}$

$\sin\theta=\frac{2}{\sqrt{29}},\cos\theta=\frac{5}{\sqrt{29}}$

$x=\frac{\sqrt{29}}{3}\times\left(\frac{5}{\sqrt{29}}\right)+4=\frac{5}{3}+4=\frac{17}{3}$

$y=\frac{\sqrt{29}}{3}\times\frac{2}{\sqrt{29}}+3=\frac{2}{3}+3=\frac{11}{3}$

$B\equiv\left(\frac{17}{3},\frac{11}{3}\right)$

Question: Find remainder when $(2024)^{2024}$ is divided by 7.

Answer: 1.00

Solution:

$(2024)^{2024}$

$=(289\times+1)^{2024}$

$= {}^{2024}C_0 + {}^{2024}C_1(289 \times 7) + ...$

Remainder = 1

Question: $z = 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$, find arg.

Answer: $\frac{3\pi}{5}$

Solution:

$z = 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$

$= 2\cos^2\frac{6\pi}{10} + 2i\sin\frac{6\pi}{10}\cos\frac{6\pi}{10}$

$= 2\cos\frac{3\pi}{5}\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$

$\arg(z) = \frac{3\pi}{5}$

Question: Sum & product of mean & variance of Binomial distribution are 24 & 128 respectively. Find probability of 1 or 2 successes.

Answer:

Solution:

Mean and variance are roots of $x^2 - 24x + 128 = 0$

$\Rightarrow$ Mean $= np = 16$

Variance $= npq = 8$

$\Rightarrow p = q = \frac{1}{2}$, $n = 32$

$p(x=1) + p(x=2) = {}^{32}C_1\left(\frac{1}{2}\right)^{32} + {}^{32}C_2\left(\frac{1}{2}\right)^{32}$

$= \left({}^{32}C_1 + {}^{32}C_2\right)\left(\frac{1}{2}\right)^{32}$

$= {}^{33}C_2\left(\frac{1}{2}\right)^{32}$

Question: Find dictionary rank of MANKIND.

Answer: 1492.00

Solution:

M A N K I N D

$\frac{4 \cdot 6!}{2!} + 0 + \frac{3(4!)}{2!} + 2(3!) + 2! + 1 + 1$

$= 1440 + 36 + 12 + 2 + 1 + 1$

$= 1492$

Question: If the locus of centre (α, β) of circle that touches both $x^2+(y-1)^2=1$ & x-axis, then find area enclosed by locus & $y=4$

Answer: $\frac{64}{3}$

Solution:

$(x-\alpha)^2+(y-\beta)^2=\beta^2$

$|\beta+1|=\sqrt{(\alpha)^2+(\beta-1)^2}$

$\Rightarrow \alpha^2=4\beta$

$\Rightarrow x^2=4y$

$\Rightarrow y=\frac{x^2}{4}$

Area $=2\int_0^4\left(4-\frac{x^2}{4}\right)=\frac{64}{3}$

Question: $p(x)=x^2+ax^2+bx+c$, $y=p(x)$ touches x-axis at $(-2,0)$ & $p'(0)=3$. Find local maxima.

Answer: 0.00

Solution:

$p(x)=x^3+ax^2+bx+c$

$p(-2)=0$

$0=-8+4a-2b+c$

$p'(-2)=0$

$\Rightarrow 12-4a+b=0$

$p'(0)=3$

$\Rightarrow c=3$

$\Rightarrow b=7$

$\Rightarrow a=\frac{19}{4}$

$p(x)=x^3+\frac{19}{4}x^2+7x+3$

$p'(x)=3x^2+\frac{19}{2}x+7$

$$x=\frac{\frac{-19}{2}\pm\sqrt{\left(\frac{19}{2}\right)^2-4\cdot 7\cdot 3}}{6}$$

$$=\frac{\frac{-19}{2}\pm\frac{5}{2}}{6}=-2,\frac{-7}{6}$$

$$p''(x) = 6x + \frac{19}{2}$$

$$p''(-2) = -12 + \frac{19}{2} < 0$$

$$p''\left(\frac{-7}{6}\right) > 0$$

Maxima = $p(-2) = 0$